ANALYTICAL APPROXIMATIONS

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho d\rho$$

in which $I_0(z)$ is the usual Bessel function. To better than .00014 over $(-\infty,\infty)$,

$$q(1,x) \doteq 1 - \frac{.3935}{\left[1+.0968x^2+.0047x^4+.00028x^6\right]^4}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the $q(R,x)$ surface for small values of R.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho d\rho$$

in which $I_0(z)$ is the usual Bessel function. To better than .0035 over $(-\infty,\infty)$,

$$q(2,x) \doteq 1 - \frac{.865}{\left[1+.038x^2+.004x^4\right]^4}.$$

The parametric form used is convenient for approximating fixed-R cross-sections of the $q(R,x)$ surface for small values of R.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho\, d\rho$$

in which $I_0(z)$ is the usual Bessel function. To better than .001 over $(-\infty, \infty)$,

$$q(2,x) \doteq 1 - \frac{.865}{\left[1+.0401x^2+.00309x^4+.000075x^6\right]^4}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the $q(R,x)$ surface for small values of R.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\,\rho\,d\rho$$

in which $I_o(z)$ is the usual Bessel function. To better than .0014 over $(-\infty, \infty)$,

$$q(1,x) \doteq 1 - \frac{.393}{\left[1+.093x^2+.007x^4\right]^4}$$

The parametric form used is convenient for approximating fixed-R cross-sections of the $q(R,x)$ surface for small values of R.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho\, d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0009 over $(0,\infty)$,

$$q(2,2+y) \doteq 1 - \frac{.397}{\left[1 + .236y + .066y^2 + .066y^3\right]^4}.$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the $q(R,R+y)$ surface for any $R \geq 0$ and for y ranging over $(0,\infty)$.

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